Plan du cours

Introduction aux

- **Continuous conduction mode (CCM)**
- **Discontinuous conduction mode (DCM)**

References additionelles:
Fundamentals of Power Electronics, Robert W. Erickson, Dragan Maksimović, SECOND EDITION University of Colorado Boulder, Colorado
Fundamentals of Power Semiconductor Devices, B. J. Baliga, Springer (2008)

Infos:

Before class Tuesday 02/12/2025 (simulation session):
Read the simulation tutorial.
Install MATLAB/Simulink with required Simscape toolboxes.
Follow the tutorial, conduct the circuit simulation as introduced.
Finish the homework in the simulation tutorial.

Note that the content in the simulation tutorial will be asked and examined for the final grade.

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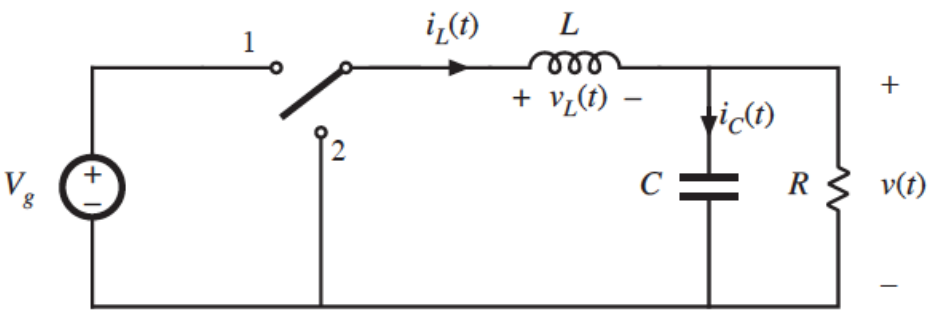
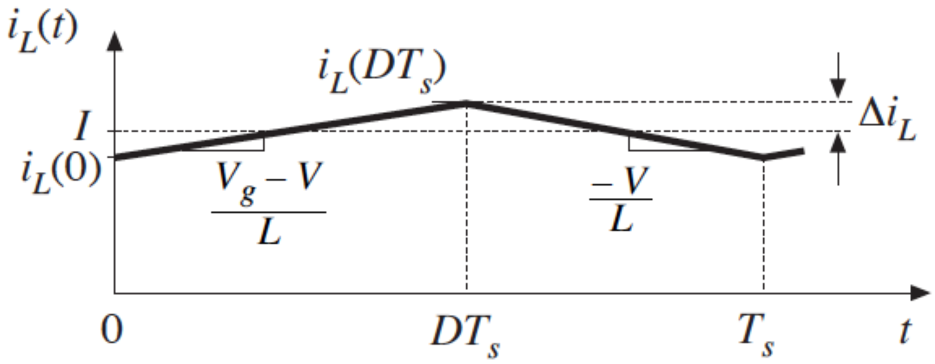
Theoretical design:
Finish the PWM circuit exercise and submit it for evaluation before Friday **05/12/2025.**

Experiments: circuit should be mounted on the bread board by the lab session Tuesday **09/12/2025**, and each group will be checked whether the circuit is ready.

We will check whether these activities were correctly executed by each group, which will compose part of your grades.

Mode de conduction continu (CCM)





$$(change\ in\ i_L) = (slope)(length\ of\ subinterval)$$

$$(2\Delta i_L) = \left(\frac{V_g - V}{L}\right)(DT_s)$$

$$\Rightarrow \qquad \Delta i_L = \frac{V_g - V}{2L} DT_s \qquad\qquad L = \frac{V_g - V}{2\Delta i_L} DT_s$$

Le courant sur l'inducteur est continu: la bobine ne se décharge jamais complètement!

Ceci est dénommé le **mode de conduction continu (ou continuous conduction mode CCM)**

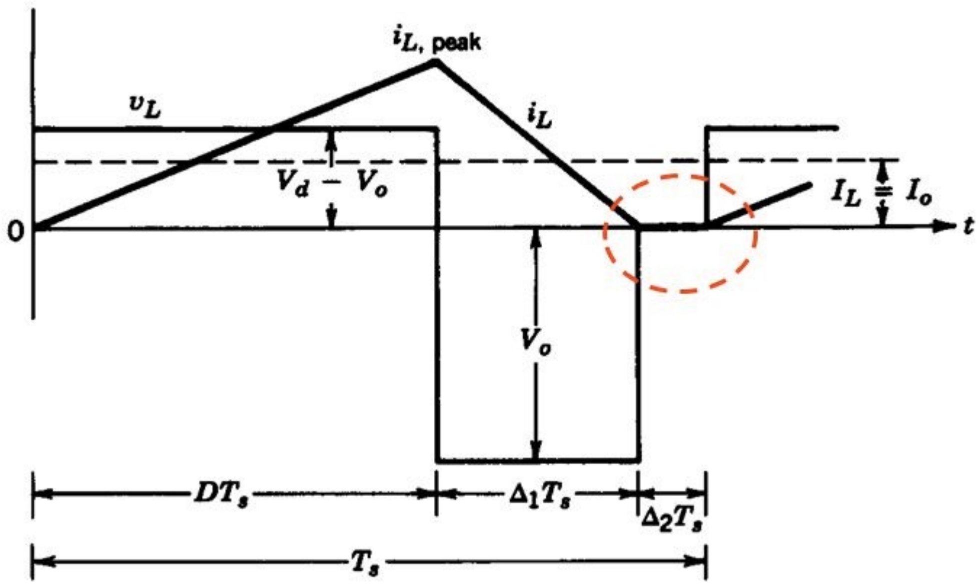
Introduction au mode de conduction discontinu (Discontinuons Conduction Mode - DCM)

Le courant traversant l'inductance s'annule pendant une partie de la période.

La seule différence avec le principe de fonctionnement décrit précédemment, est que l'inductance est complètement déchargée au début du cycle.

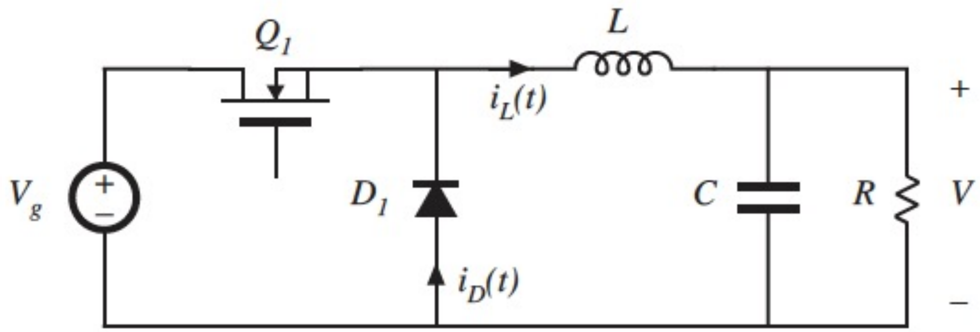
Bien que faible, la différence entre conduction continue et discontinue a un fort impact sur la formule de la tension de sortie. En plus, le **facteur de conversion M** devient **dépendent de la charge.**

Cela se passe typiquement aux convertisseurs DC-DC avec charges faibles: résistance élevée

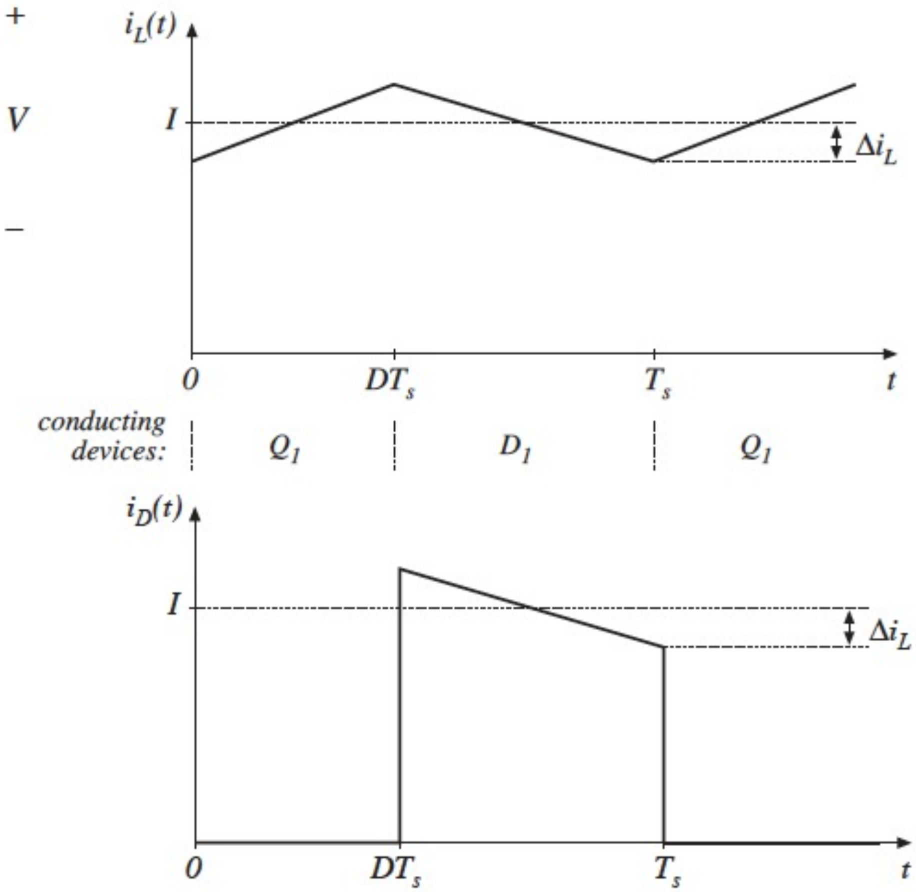


Introduction au mode de conduction discontinu (DCM)

Buck converter example, with single-quadrant switches



continuous conduction mode (CCM)



Minimum diode current is $(I - \Delta i_L)$

Dc component $I = V/R$

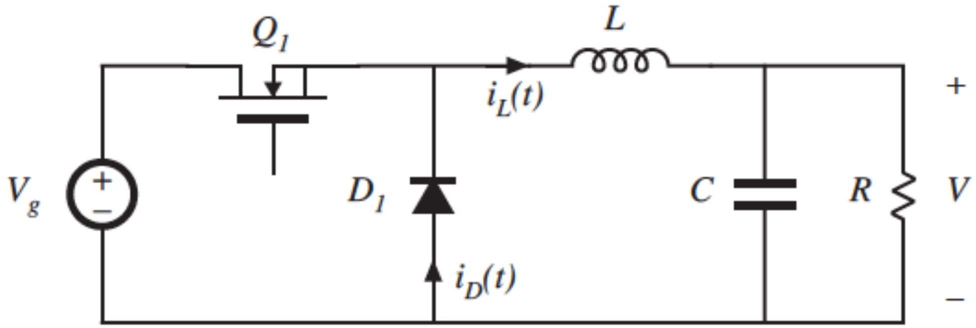
Current ripple is

$$\Delta i_L = \frac{(V_g - V)}{2L} DT_s = \frac{V_g DD'T_s}{2L}$$

Note that I depends on load, but Δi_L does not.

Introduction au mode de conduction discontinu (DCM)

Increase R, until $I = \Delta i_L$

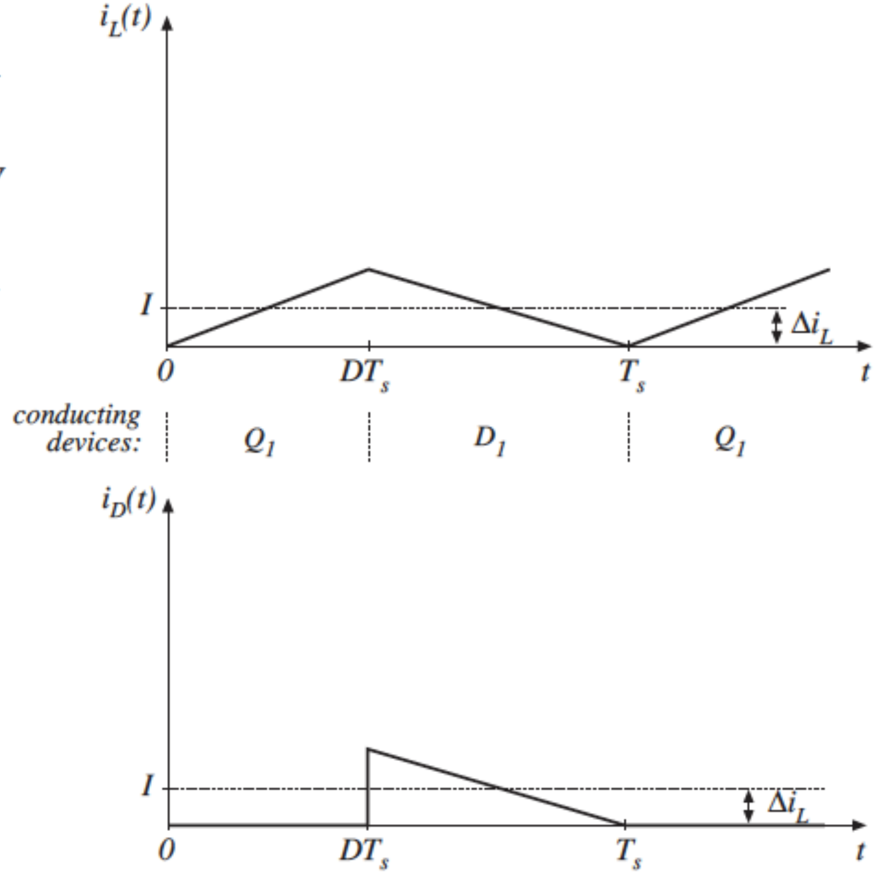


Minimum diode current is $(I - \Delta i_L)$
Dc component $I = V/R$
Current ripple is

$$\Delta i_L = \frac{(V_g - V)}{2L} DT_s = \frac{V_g DD'T_s}{2L}$$

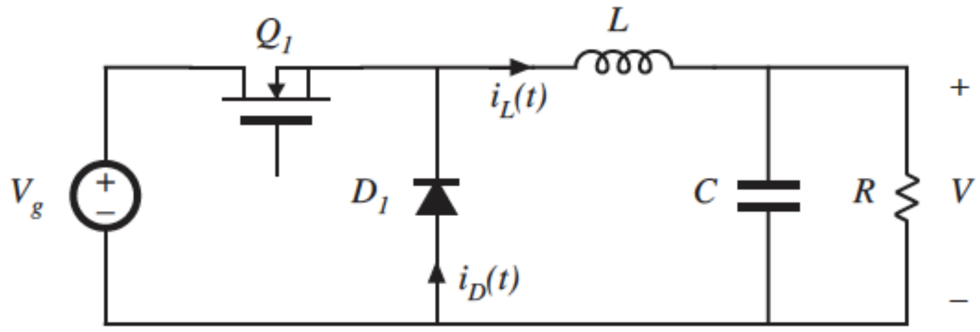
Note that I depends on load, but Δi_L does not.

CCM-DCM boundary



Introduction au mode de conduction discontinu (DCM)

Increase R some more, such that $I < \Delta i_L$

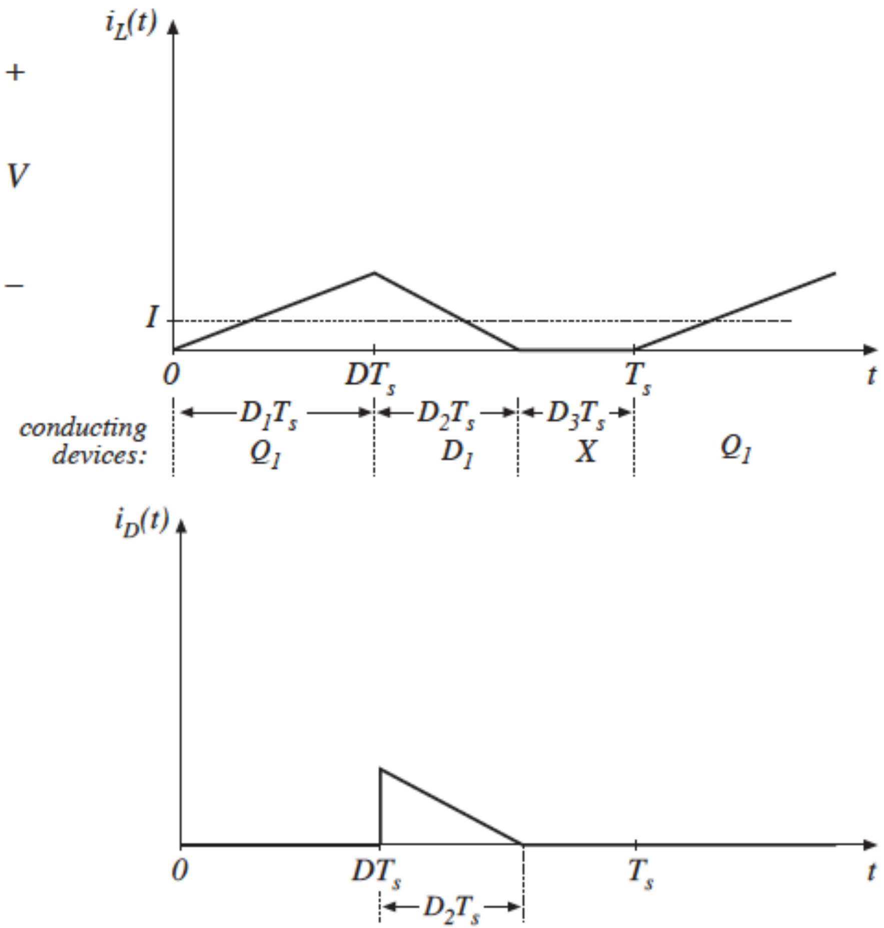


Minimum diode current is $(I - \Delta i_L)$
Dc component $I = V/R$
Current ripple is

$$\Delta i_L = \frac{(V_g - V)}{2L} DT_s = \frac{V_g DD'T_s}{2L}$$

Note that I depends on load, but Δi_L does not.
The load current continues to be positive and non-zero.

Discontinuous conduction mode



Introduction au mode de conduction discontinu (DCM)

$$I > \Delta i_L \quad \text{for } CCM$$
$$I < \Delta i_L \quad \text{for } DCM$$

Insert buck converter expressions for I and Δi_L :

$$\frac{DV_g}{R} < \frac{DD' T_s V_g}{2L}$$

Simplify:

$$\frac{2L}{RT_s} < D'$$

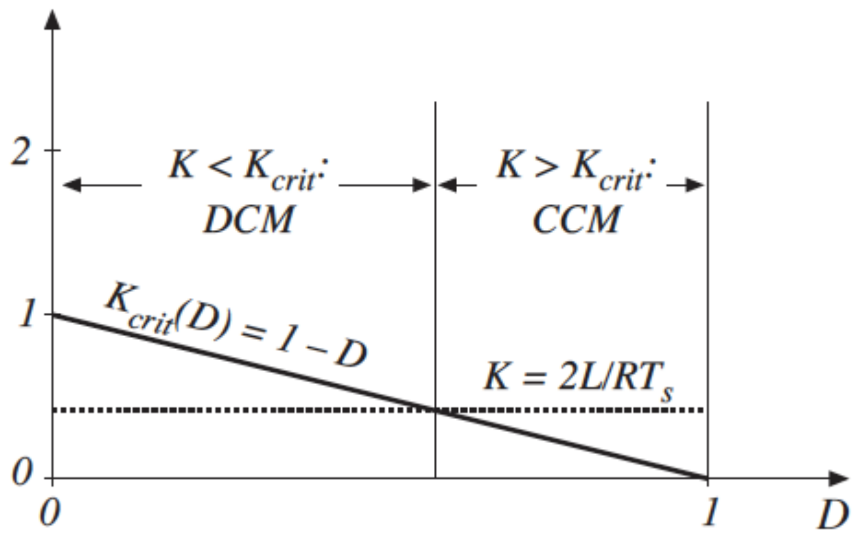
This expression is of the form

$$K < K_{crit}(D) \qquad \text{for } DCM$$

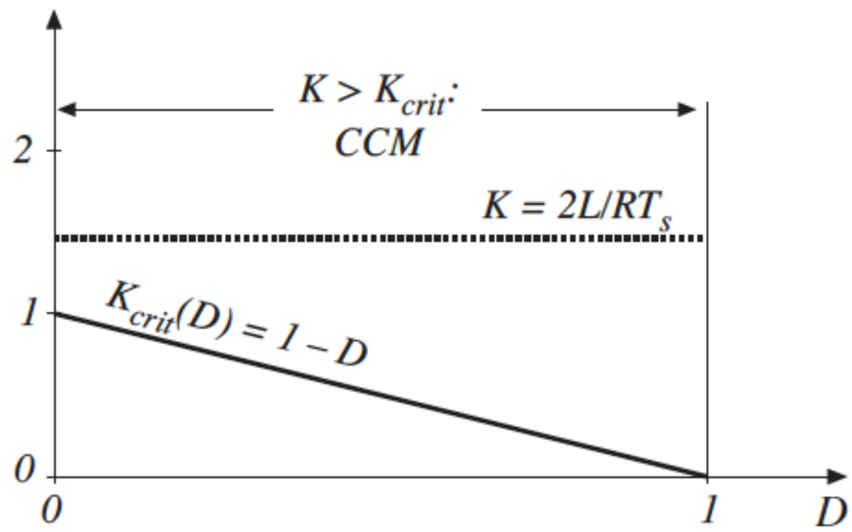
$$\text{where} \quad K = \frac{2L}{RT_s} \quad \text{and} \quad K_{crit}(D) = D'$$

Introduction au mode de conduction discontinu (DCM)

for $K < 1$:



for $K > 1$:



Solve K_{crit} equation for load resistance R:

$$R < R_{crit}(D) \quad \textit{for CCM}$$

$$R > R_{crit}(D) \quad \textit{for DCM}$$

where $$R_{crit}(D) = \frac{2L}{D'T_s}$$

Résumé

$$K > K_{crit}(D) \quad or \quad R < R_{crit}(D) \quad for\ CCM$$
$$K < K_{crit}(D) \quad or \quad R > R_{crit}(D) \quad for\ DCM$$

Table 5.1. CCM-DCM mode boundaries for the buck, boost, and buck-boost converters

Converter	$K_{crit}(D)$	$\max_{0 \le D \le 1}(K_{crit})$	$R_{crit}(D)$	$\min_{0 \le D \le 1}(R_{crit})$
Buck	$(1 - D)$	1	$\frac{2L}{(1-D)T_s}$	$2\frac{L}{T_s}$
Boost	$D(1 - D)^2$	$\frac{4}{27}$	$\frac{2L}{D(1-D)^2 T_s}$	$\frac{27}{2}\frac{L}{T_s}$
Buck-boost	$(1 - D)^2$	1	$\frac{2L}{(1-D)^2 T_s}$	$2\frac{L}{T_s}$

DCM: discontinuous conduction mode

CCM: continuous conduction mode

EPFL

Analyse du mode de conduction discontinu

Inductor volt-second balance

$$\langle v_L \rangle = \frac{1}{T_s} \int_0^{T_s} v_L(t)\, dt = 0$$

Capacitor charge balance

$$\langle i_C \rangle = \frac{1}{T_s} \int_0^{T_s} i_C(t)\, dt = 0$$

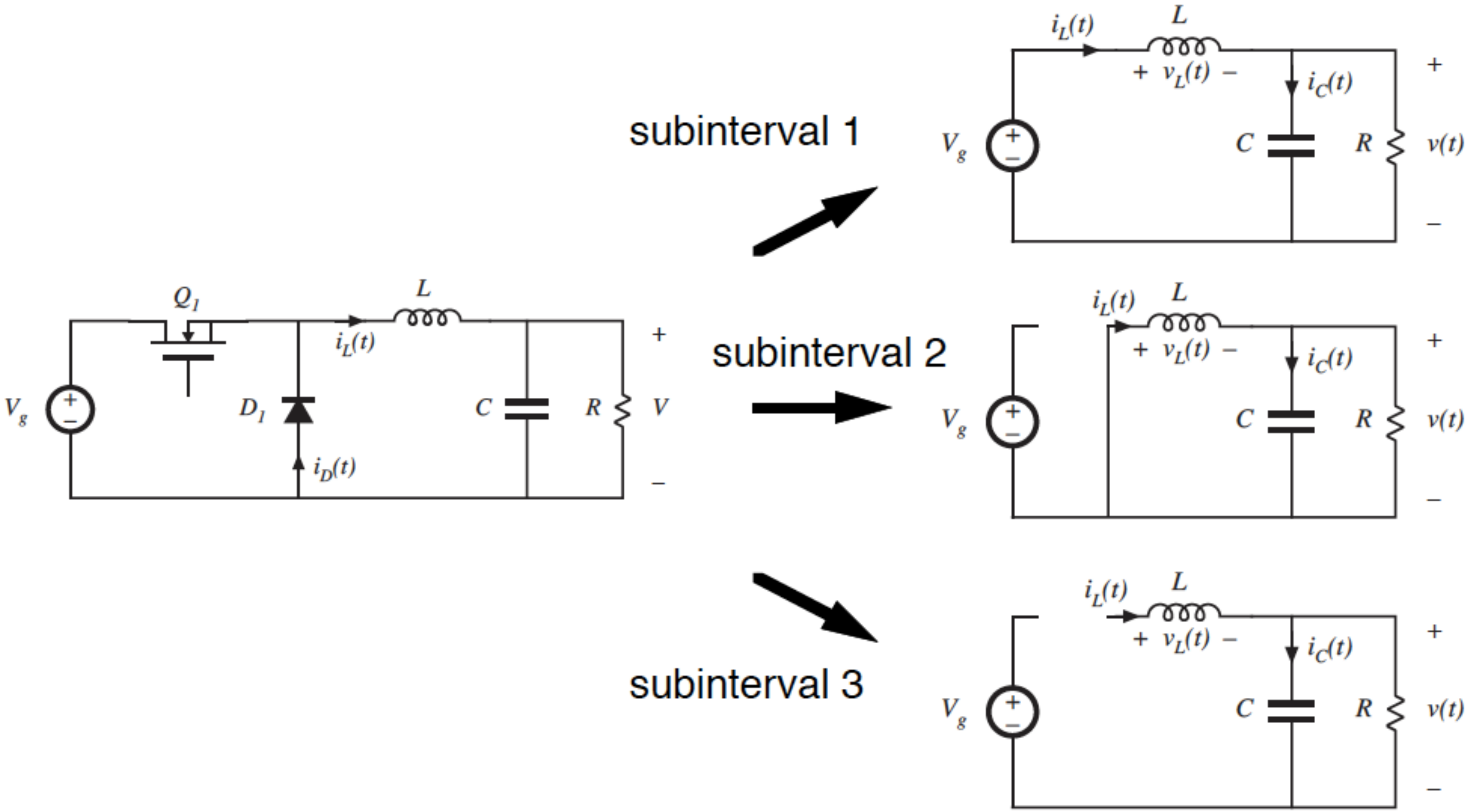
Small ripple approximation sometimes applies:

$$v(t) \approx V \quad \text{because} \quad \Delta v \ll V$$

$$i(t) \approx I \ \text{ is a poor approximation when } \ \Delta i > I$$

Converter steady-state equations obtained via charge balance on each capacitor and volt-second balance on each inductor. Use care in applying small ripple approximation.

Buck converter

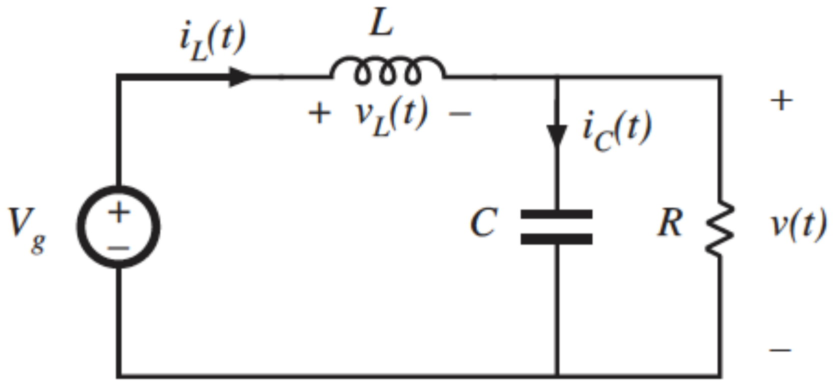


Position 1

$$v_L(t) = V_g - v(t)$$
$$i_C(t) = i_L(t) - v(t) / R$$

Small ripple approximation for $v(t)$ (but not for $i(t)$!):

$$v_L(t) \approx V_g - V$$
$$i_C(t) \approx i_L(t) - V / R$$

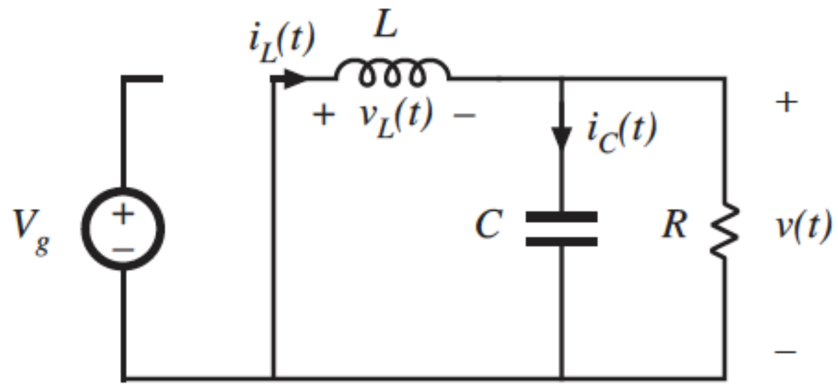


Position 2

$$v_L(t) = -\,v(t)$$
$$i_C(t) = i_L(t) - v(t)\,/\,R$$

Small ripple approximation for $v(t)$ but not for $i(t)$:

$$v_L(t) \approx -\,V$$
$$i_C(t) \approx i_L(t) - V\,/\,R$$

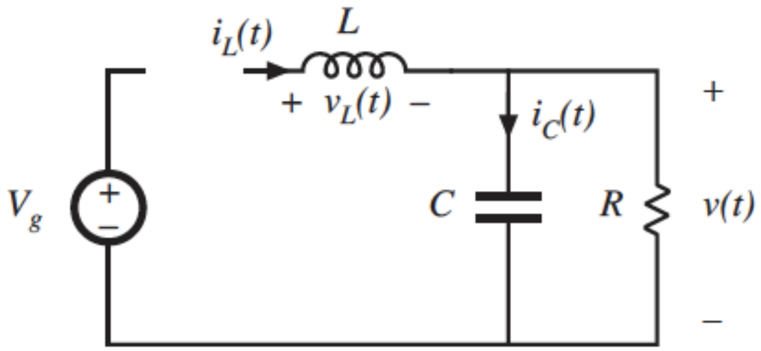


Position 3

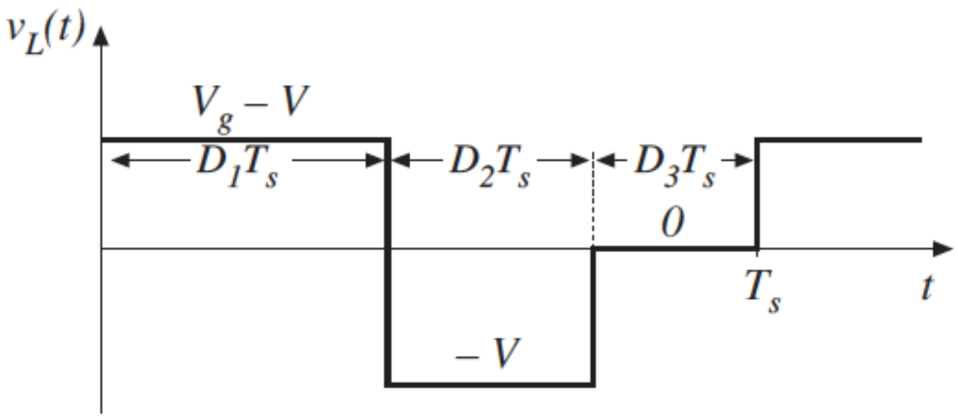
$$v_L = 0, \quad i_L = 0$$
$$i_C(t) = i_L(t) - v(t) / R$$

Small ripple approximation:

$$v_L(t) = 0$$
$$i_C(t) = -V / R$$



Inductor volt-second balance



Volt-second balance:

$$\left\langle v_L(t) \right\rangle = D_1(V_g - V) + D_2(-V) + D_3(0) = 0$$

Solve for V:

$$V = V_g \frac{D_1}{D_1 + D_2}$$

note that D_2 is unknown

Capacitor-charge balance

node equation:

$$i_L(t) = i_C(t) + V / R$$

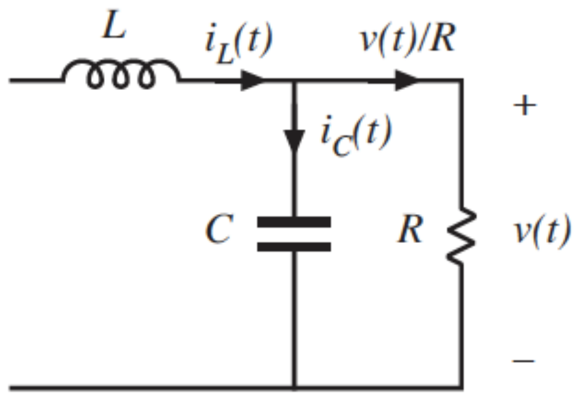
capacitor charge balance:

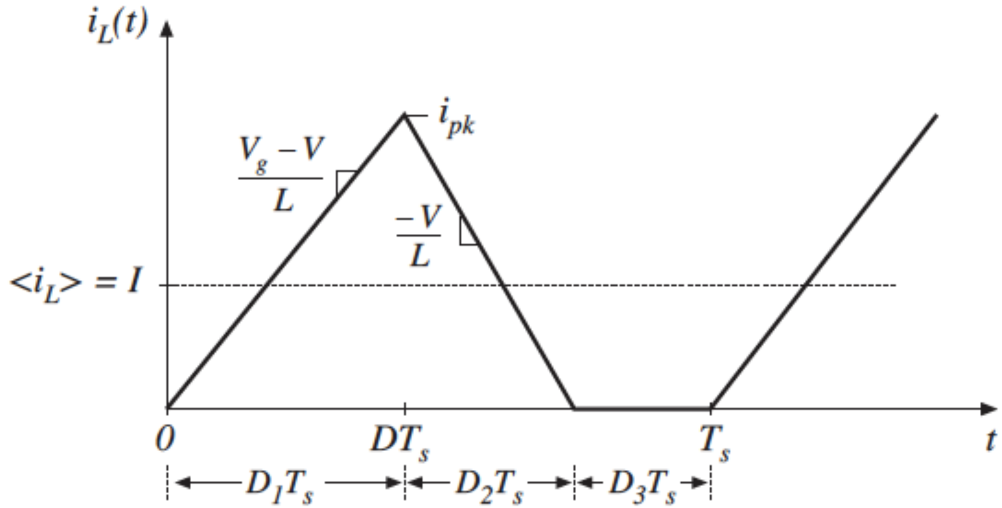
$$\langle i_C \rangle = 0$$

hence

$$\langle i_L \rangle = V / R$$

must compute dc component of inductor current and equate to load current (for this buck converter example)





Capacitor-charge balance

peak current:

$$i_L(D_1T_s) = i_{pk} = \frac{V_g - V}{L} D_1T_s$$

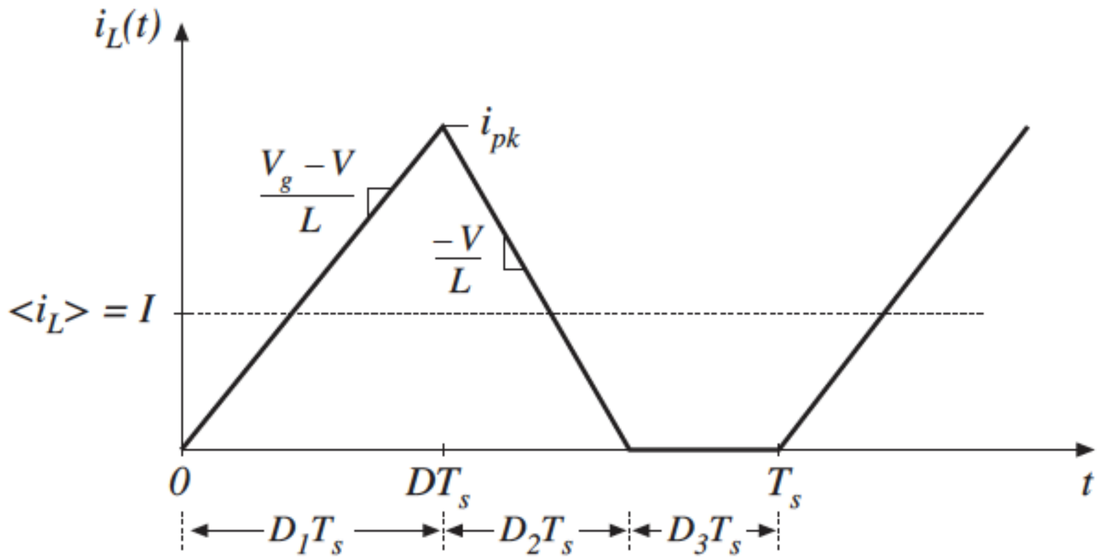
average current:

$$\langle i_L \rangle = \frac{1}{T_s}\int_0^{T_s} i_L(t)\, dt$$

triangle area formula:

$$\int_0^{T_s} i_L(t)\, dt = \frac{1}{2}\, i_{pk}\, (D_1 + D_2)T_s$$

$$\langle i_L \rangle = (V_g - V)\, \frac{D_1T_s}{2L}\, (D_1 + D_2)$$



equate dc component to dc load current:

$$\frac{V}{R} = \frac{D_1T_s}{2L}\,(D_1 + D_2)\,(V_g - V)$$

Capacitor-charge balance

Two equations and two unknowns (V and D_2):

$$V = V_g \frac{D_1}{D_1 + D_2}$$ (from inductor volt-second balance)

$$\frac{V}{R} = \frac{D_1 T_s}{2L} (D_1 + D_2)(V_g - V)$$ (from capacitor charge balance)

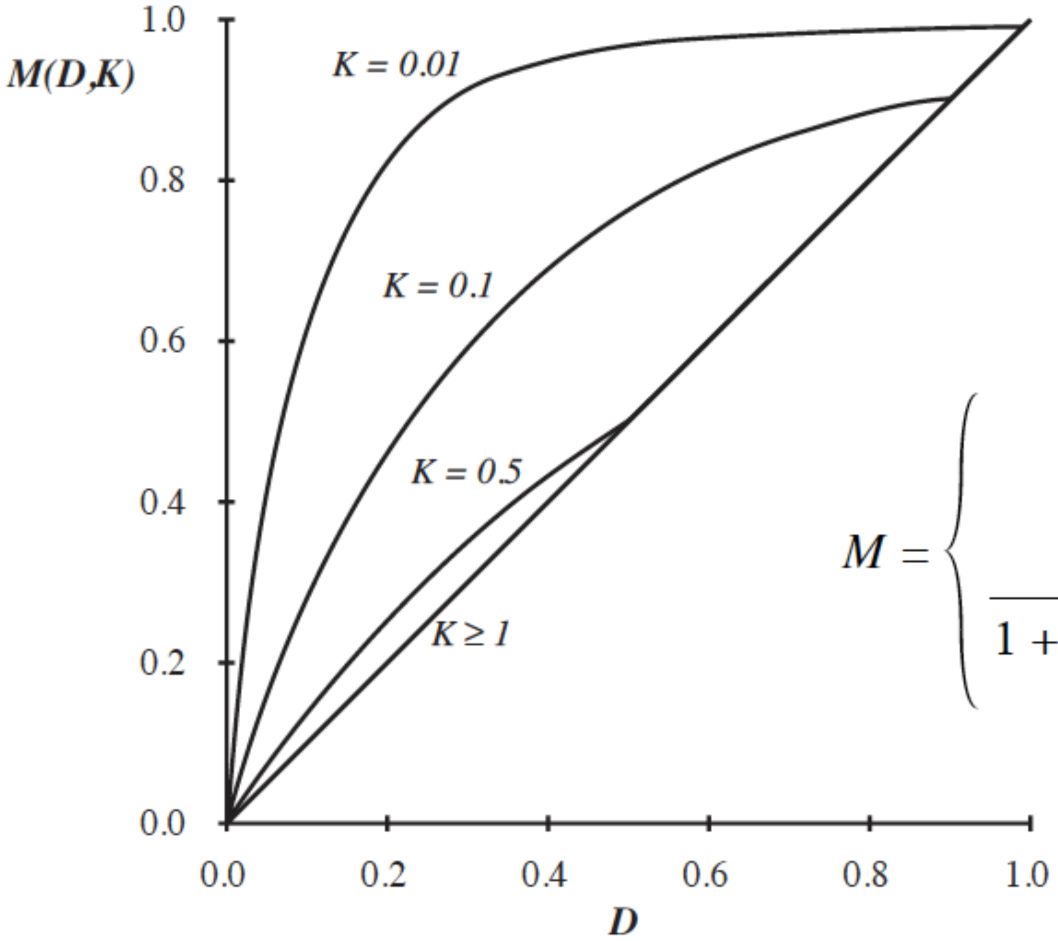
Eliminate D_2 , solve for V :

$$\frac{V}{V_g} = \frac{2}{1 + \sqrt{1 + 4K / D_1^2}}$$

where $K = 2L / RT_s$

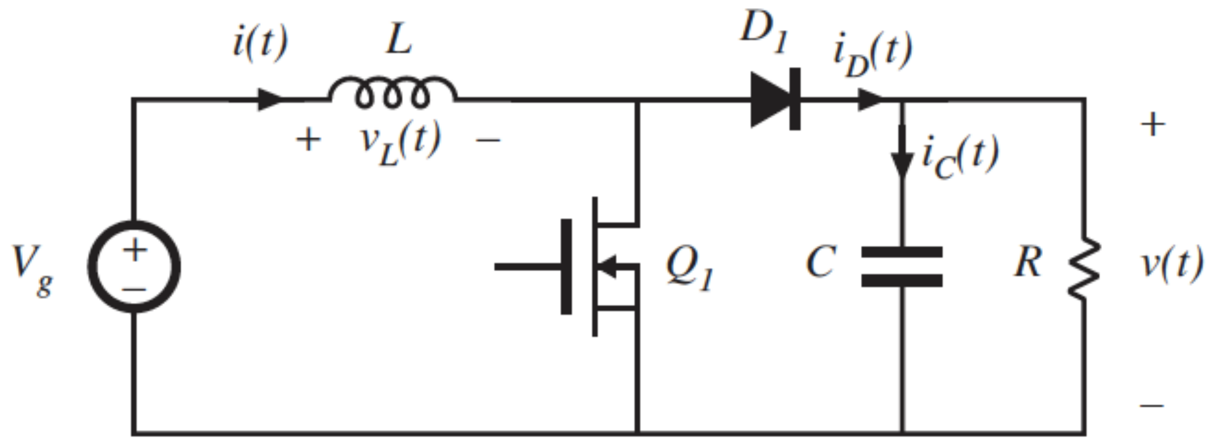
valid for $K < K_{crit}$

Buck converter



$$M = \begin{cases} D & \text{for } K > K_{crit} \\ \dfrac{2}{1 + \sqrt{1 + 4K / D^2}} & \text{for } K < K_{crit} \end{cases}$$

Boost-converter en CCM et DCM



Mode boundary:

$$I > \Delta i_L \quad \textit{for CCM}$$
$$I < \Delta i_L \quad \textit{for DCM}$$

Previous CCM soln:

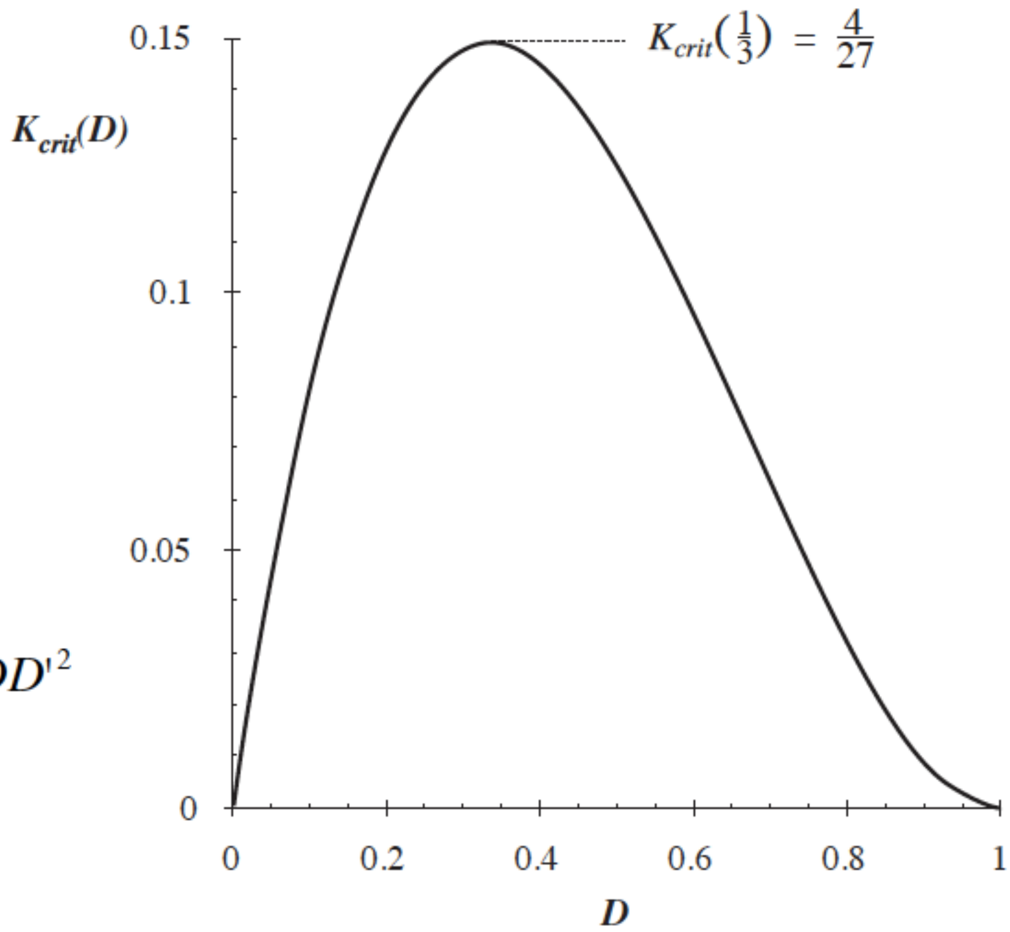
$$I = \frac{V_g}{D'^2 R} \qquad \Delta i_L = \frac{V_g}{2L} D T_s$$

Boost-converter

$$\frac{V_g}{D'^2 R} > \frac{D T_s V_g}{2L} \quad \textit{for CCM}$$

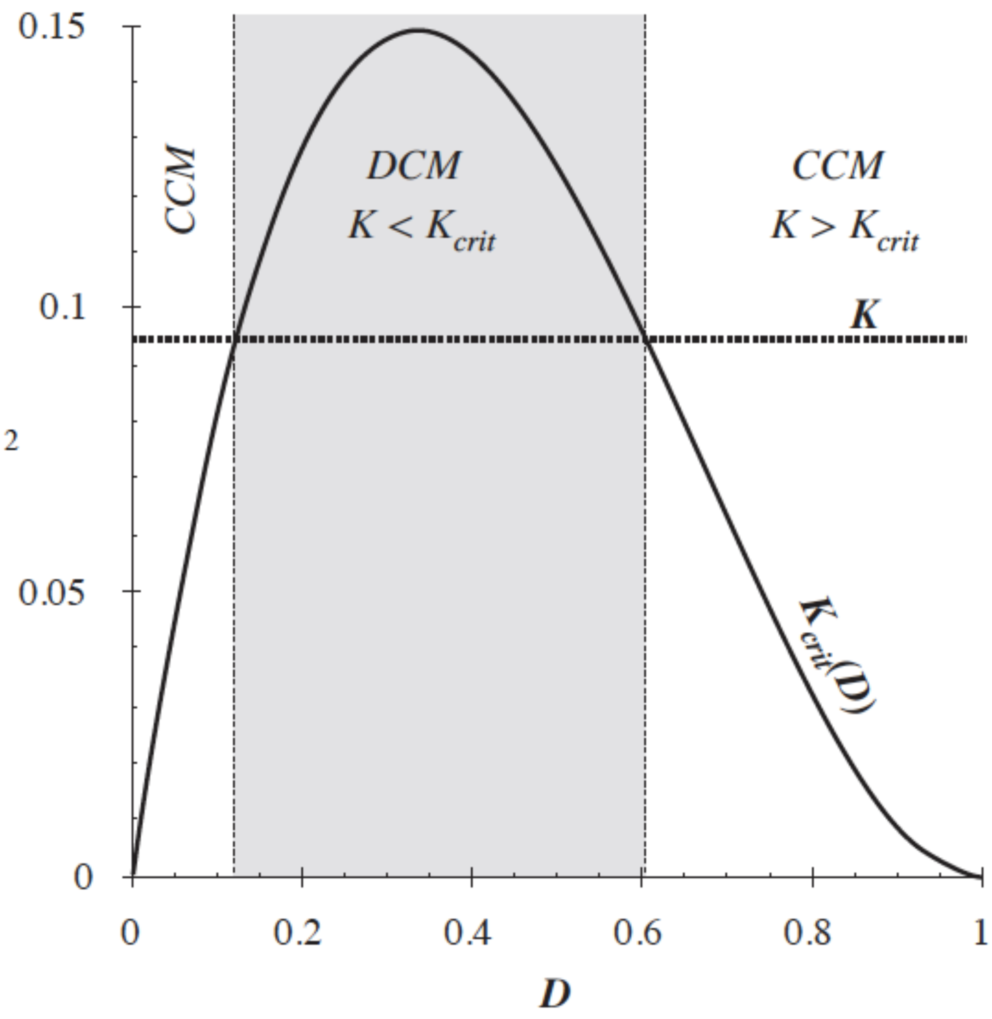
$$\frac{2L}{RT_s} > DD'^2 \quad \textit{for CCM}$$

$$K > K_{crit}(D) \quad \textit{for CCM}$$
$$K < K_{crit}(D) \quad \textit{for DCM}$$
$$\textit{where} \quad K = \frac{2L}{RT_s} \quad \textit{and} \quad K_{crit}(D) = DD'^2$$

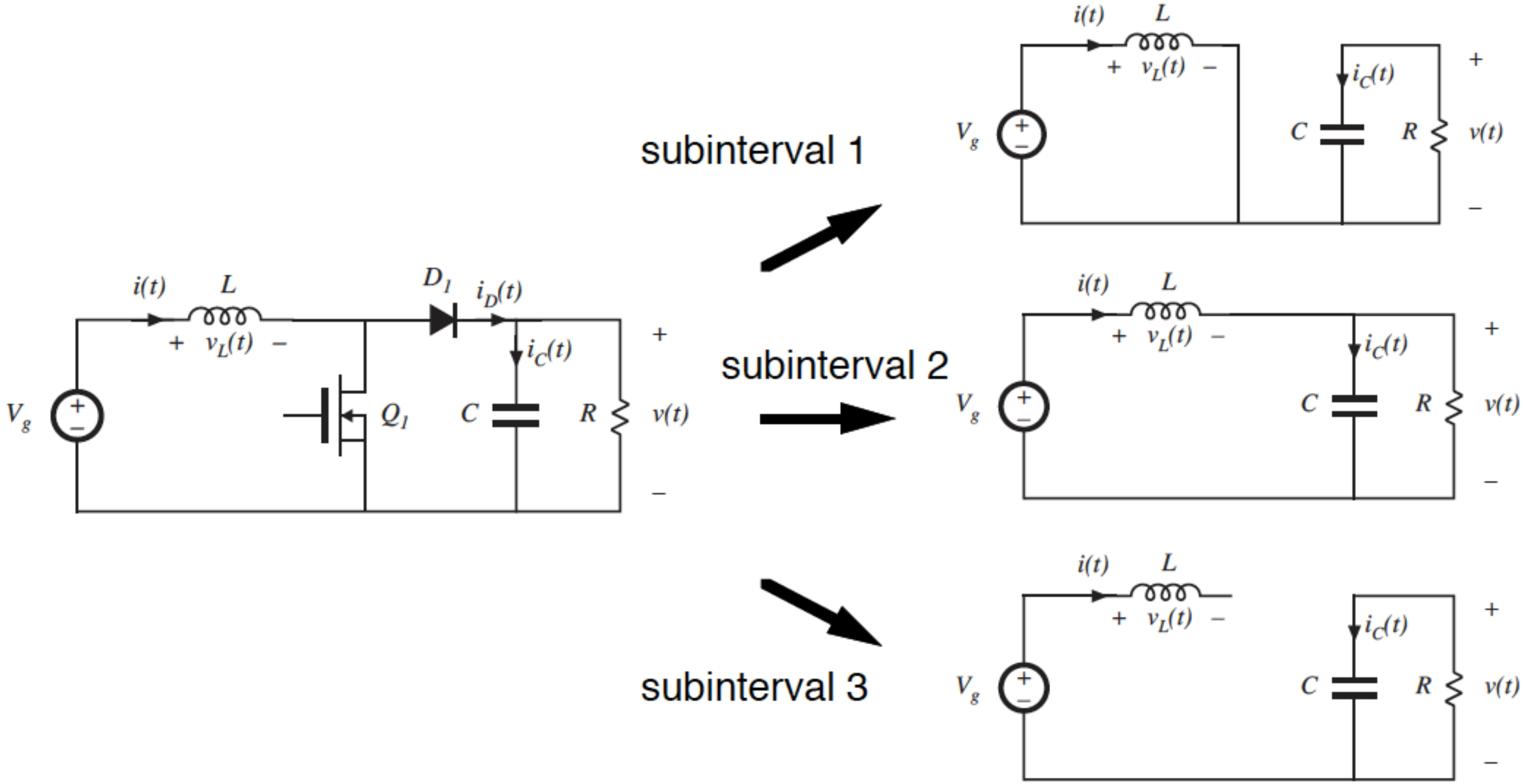


Boost-converter

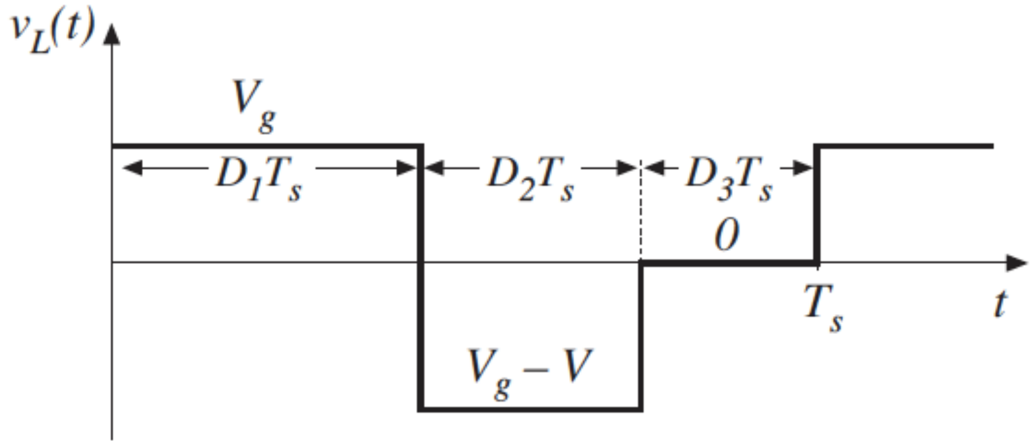
$$K > K_{crit}(D) \quad \text{for } CCM$$
$$K < K_{crit}(D) \quad \text{for } DCM$$
$$\text{where} \quad K = \frac{2L}{RT_s} \quad \text{and} \quad K_{crit}(D) = DD'^2$$



Boost-converter



Inductor volt-second balance



Volt-second balance:

$$D_1 V_g + D_2(V_g - V) + D_3(0) = 0$$

Solve for V:

$$V = \frac{D_1 + D_2}{D_2} V_g$$

note that D_2 is unknown

Capacitor charge balance

node equation:

$$i_D(t) = i_C(t) + v(t) / R$$

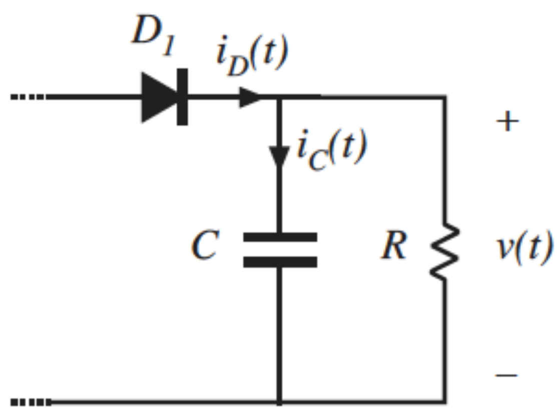
capacitor charge balance:

$$\langle i_C \rangle = 0$$

hence

$$\langle i_D \rangle = V / R$$

must compute dc component of diode current and equate to load current (for this boost converter example)



Inductor and diode curves

peak current:

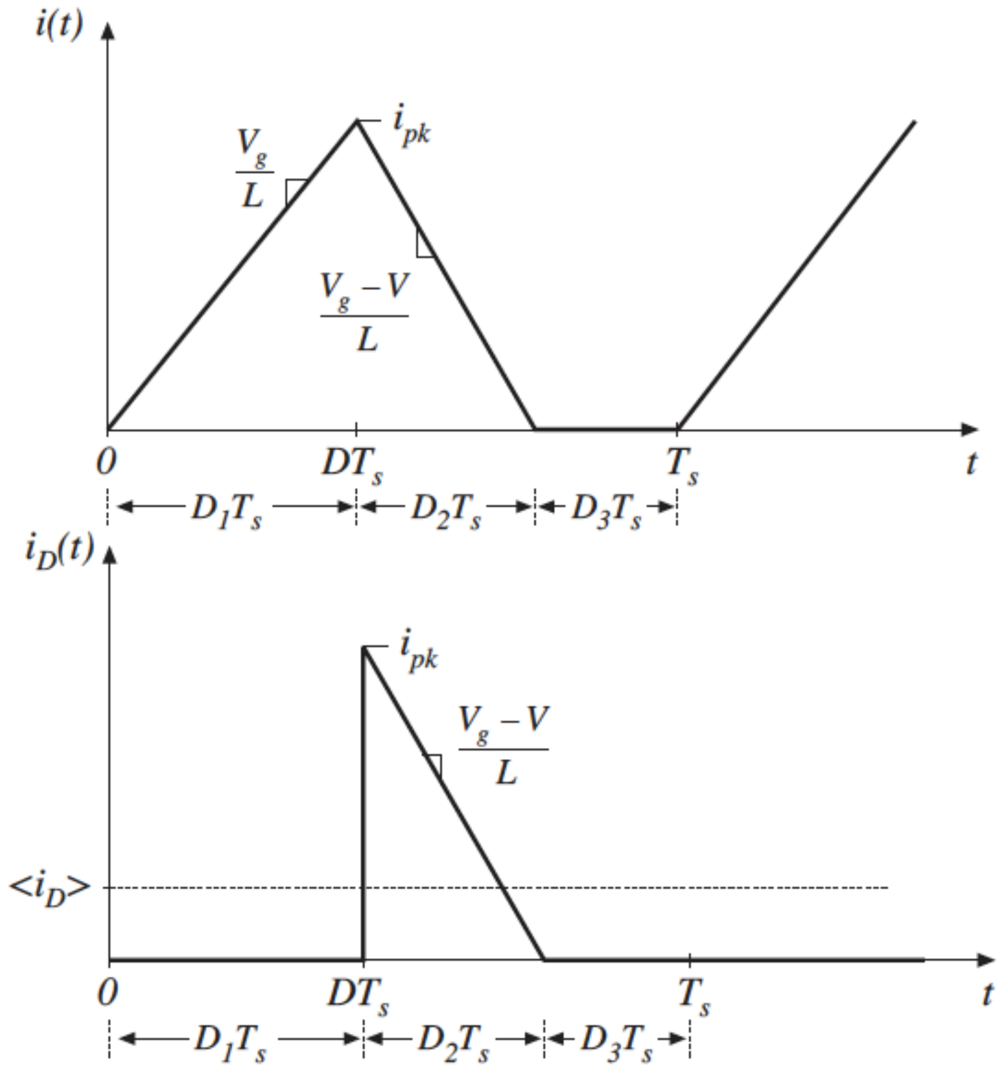
$$i_{pk} = \frac{V_g}{L} D_1 T_s$$

average diode current:

$$\langle i_D \rangle = \frac{1}{T_s} \int_0^{T_s} i_D(t)\, dt$$

triangle area formula:

$$\int_0^{T_s} i_D(t)\, dt = \frac{1}{2}\, i_{pk}\, D_2 T_s$$



Inductor and diode curves

average diode current:

$$\langle i_D \rangle = \frac{1}{T_s}\left(\frac{1}{2}\, i_{pk}\, D_2 T_s\right) = \frac{V_g D_1 D_2 T_s}{2L}$$

equate to dc load current:

$$\frac{V_g D_1 D_2 T_s}{2L} = \frac{V}{R}$$

Inductor and diode curves

Two equations and two unknowns (V and D_2):

$$V = \frac{D_1 + D_2}{D_2} V_g \qquad \text{(from inductor volt-second balance)}$$

$$\frac{V_g D_1 D_2 T_s}{2L} = \frac{V}{R} \qquad \text{(from capacitor charge balance)}$$

Eliminate D_2 , solve for V. From volt-sec balance eqn:

$$D_2 = D_1 \frac{V_g}{V - V_g}$$

Substitute into charge balance eqn, rearrange terms:

$$V^2 - VV_g - \frac{V_g^2 D_1^2}{K} = 0$$

Solution

$$V^2 - VV_g - \frac{V_g^2 D_1^2}{K} = 0$$

Use quadratic formula:

$$\frac{V}{V_g} = \frac{1 \pm \sqrt{1 + 4D_1^2 / K}}{2}$$

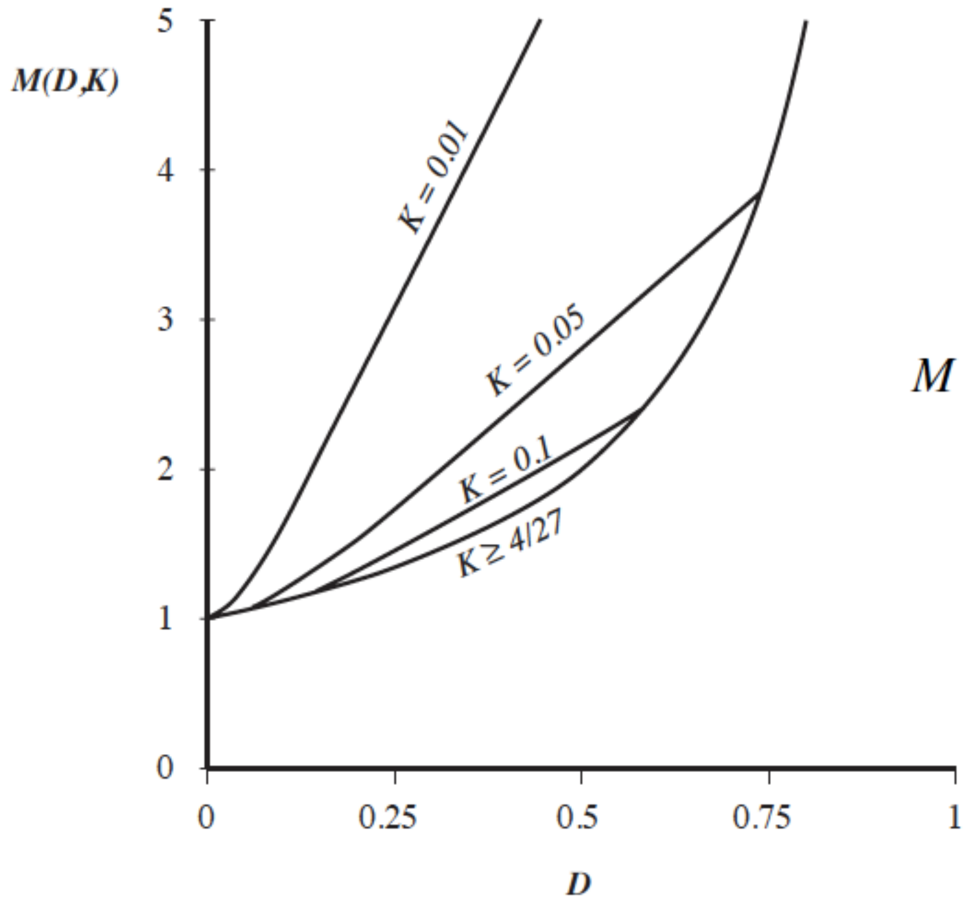
Note that one root leads to positive V, while other leads to negative V. Select positive root:

$$\frac{V}{V_g} = M(D_1, K) = \frac{1 + \sqrt{1 + 4D_1^2 / K}}{2}$$

where $K = 2L / RT_s$
valid for $K < K_{crit}(D)$

Transistor duty cycle D = interval *1* duty cycle D_1

Boost converter characteristics



$$M = \begin{cases} \dfrac{1}{1-D} & \text{for } K > K_{crit} \\ \dfrac{1+\sqrt{1+4D^2/K}}{2} & \text{for } K < K_{crit} \end{cases}$$

Approximate M in DCM:

$$M \approx \frac{1}{2} + \frac{D}{\sqrt{K}}$$

Résumé

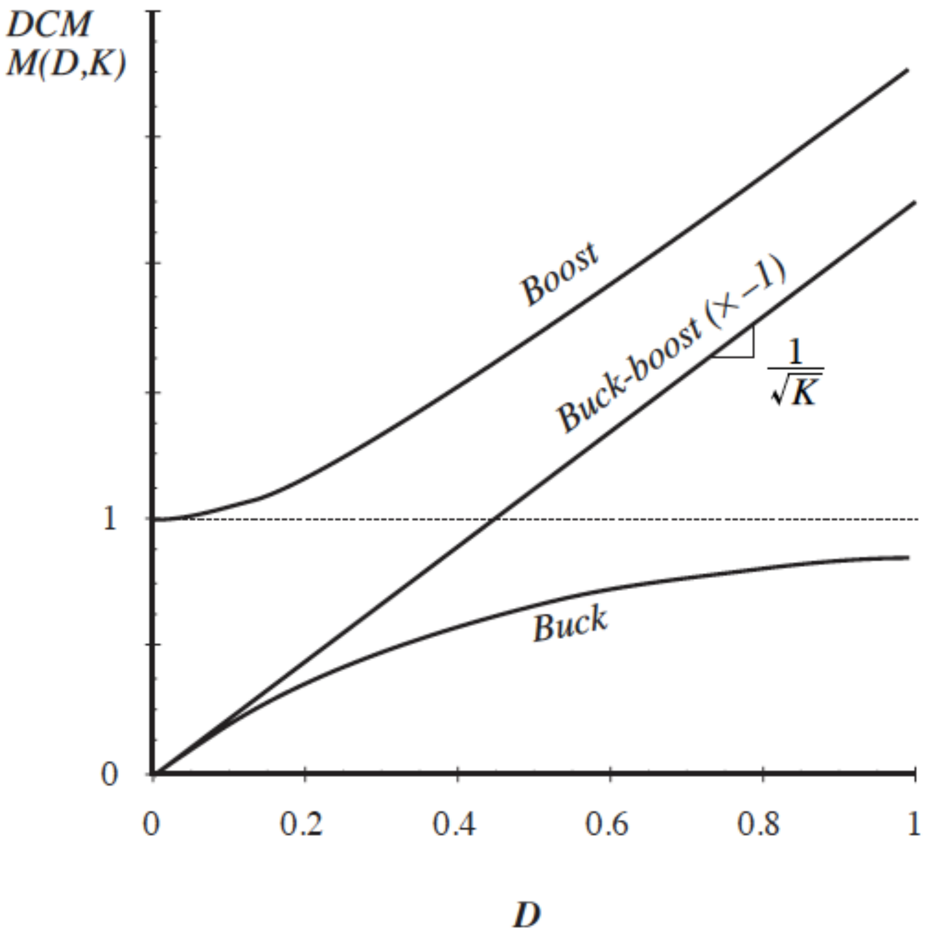
Converter	$K_{crit}(D)$	DCM $M(D,K)$	DCM $D_2(D,K)$	CCM $M(D)$
Buck	$(1 - D)$	$\frac{2}{1+\sqrt{1+4K/D^2}}$	$\frac{K}{D}M(D,K)$	D
Boost	$D(1-D)^2$	$\frac{1+\sqrt{1+4D^2/K}}{2}$	$\frac{K}{D}M(D,K)$	$\frac{1}{1-D}$
Buck-boost	$(1-D)^2$	$-\frac{D}{\sqrt{K}}$	$\sqrt{K}$	$-\frac{D}{1-D}$

with $K = 2L/RT_s$. *DCM occurs for* $K < K_{crit}$.

DCM: discontinuous conduction mode

CCM: continuous conduction mode

Résumé des caractéristiques du mode de conduction discontinu (DCM)



- DCM buck and boost characteristics are asymptotic to $M = 1$ and to the DCM buck-boost characteristic
- DCM buck-boost characteristic is linear
- CCM and DCM characteristics intersect at mode boundary. Actual M follows characteristic having larger magnitude
- DCM boost characteristic is nearly linear

Résumé des caractéristiques du DCM

1. The discontinuous conduction mode occurs in converters containing current- or voltage-unidirectional switches, when the **inductor current or capacitor voltage ripple is large enough to cause the switch current or voltage to reverse polarity**.

2. Conditions for operation in the discontinuous conduction mode can be found by determining when the inductor current or capacitor voltage ripples and dc components cause the switch on-state current or off-state voltage to reverse polarity.

3. The dc conversion ratio M of converters operating in the discontinuous conduction mode can be found by application of the principles of inductor volt-second and capacitor charge balance.

4. **Extra care is required when applying the small-ripple approximation**. Some waveforms, such as the output voltage, should have small ripple which can be neglected. Other waveforms, such as one or more **inductor currents, may have large ripple that cannot be ignored.**

5. The characteristics of a converter changes significantly when the converter enters DCM. The output voltage becomes load dependent, resulting in an increase in the converter output impedance.